

## RESEARCH ARTICLE

# Beamforming and power allocation in the downlink of MIMO cognitive radio systems based on multiobjective optimization

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**Summary**

In this paper, power allocation and beamforming are considered in a multiple input multiple output (MIMO) downlink cognitive radio (CR) communication system, which a base station (BS) serves one primary user (PU) and one secondary user (SU). In order to design the CR system, a constrained multiobjective optimization problem is presented. Two objectives are the signal to noise plus interference ratios (SINRs) of PU and SU. Since PU has a spectrum license for data communication, a constraint in the optimization problem is that the SINR of PU must be greater than a predefined threshold based on the PU demand requirement. Another constraint is a limitation on power in BS. By considering the mentioned model, three iterative algorithms are proposed. At each iteration of all algorithms, the receiver beamforming vectors are derived based on the maximization of PU and SU SINRs, by assuming that the allocated powers and BS beamforming vectors are known. Also, power is assigned to users such that the constraint of power limitation is satisfied. The difference between the algorithms is in the obtaining of transmitter beamforming parameters. We evaluate the performance of the proposed algorithms in terms of bit error rate (BER) in simulations. Also, the computational complexity of the proposed algorithms is obtained.

**KEYWORDS**

beamforming, cognitive radio, MIMO, optimization problem, power allocation

## 1 | INTRODUCTION

Due to bandwidth scarcity, cognitive radio (CR) systems are introduced to increase spectrum efficiency. The main idea of CR is the coexistence of primary users (PUs) and secondary users (SUs).<sup>1-4</sup> The former users have the spectrum access license and high priority to use the bandwidth. SUs can access the frequency bands of PU in conditions of normal PU communication is not affected. In order to serve SUs, three scenarios, namely, underlay, interweave, and overlay, have been proposed.<sup>5-7</sup> In the underlay method, SUs coexist with PUs, where the interference from SU transmitters to PU receivers are limited. In an interweave mode, SUs transmit their data in the spectrum holes of PU frequency bands. Thus, SUs should sense the spectrum to detect free channels. In the overlay model, SUs cooperate with PUs by relaying the data of PUs, in order to get chance of data transmission. In this paper, we consider the underlay model.

On the other hand, by using multiple input multiple output (MIMO) techniques, spatial diversity or spatial multiplexing can be achieved, which leads to improving the performance of communication systems.<sup>8,9</sup> Therefore, CR and MIMO have been combined in order to benefit the capacity of multiple antennas.<sup>10-12</sup> An efficient way to achieve spatial multiplexing and limit interference in MIMO CR systems is beamforming of transceiver antennas.<sup>13,14</sup> Besides, an effective power allocation technique can remove or reduce the interference induced from SU to PU.<sup>15</sup> As a result in recent years, joint power allocation and beamforming has been widely studied in the literature for CR systems.<sup>13,14</sup>

Beamforming and power allocation of MIMO CR studies can be categorized into two classes. In one category, some separate pair users communicate with each other in the presence of other pair users such as previous studies.<sup>15-17</sup> In another class, one or more base stations (BSs) serve PUs and SUs in downlink or uplink.<sup>18-21</sup> In this paper, downlink of the second class is considered. In almost studies of the second category, a BS serves SUs and another BS serves PUs. For example, we can point to other studies.<sup>18,20,22</sup> Therefore, it is needed to implement two related network structures, which these networks generate interference on each other. For reducing implementation cost, we assume just one BS transmits data to both PUs and SUs.

Another classification can be done on the downlink CR systems based on the number of used antennas. In many studies, BSs are equipped with multiple antennas while in user receiver sides, just a single antennas is applied (multiple input single output [MISO]).<sup>20,22,23</sup> In other papers, multiple antennas are utilized in BS and user sides.<sup>18,19</sup> In here, in order to achieve both transmitter and receiver diversities and limit interference effectively, MIMO is used.

In Zamiri-Jafarian Hossein and Jannat-Abad,<sup>19</sup> an effective method is proposed for power allocation and beamforming, in a downlink MIMO CR network. In this system, a BS serves one PU and several SUs. In order to obtain the BS beamforming vectors of PU and SUs, the total signal to interference plus noise ratios (TSINRs) of SUs are maximized by supposing to know all parameters of user receivers. Also, the receiver beamforming vectors of SUs are derived by maximizing TSINR and knowing all other parameters. Besides, the receiver beamforming vector of PU is extracted such that a constraint on PU SINR is satisfied. Finally, an iterative algorithm is proposed in Zamiri-Jafarian Hossein and Jannat-Abad,<sup>19</sup> which the authors of this paper call it cooperative beamforming and power allocation (CBPA) algorithm. In Zamiri-Jafarian Hossein and Jannat-Abad,<sup>19</sup> the PU SINR improvement is not considered. Also, optimization of TSINR does not lead to maximize the SINR of each SU. Therefore in here, we study on CBPA algorithm improvement.

It must be noted that in recent years, a lot of other methods have been proposed in the category of resource allocation of CR systems, for example, extending CR networks to orthogonal frequency division multiplexing (OFDM)<sup>11,24,25</sup> or applying massive MIMO in this system.<sup>26-28</sup> But in this paper, we address a basic network, which as mentioned before, just a BS is used and MIMO is applied in the transceivers. In other words, the CR network model, which is presented in this paper, is investigated very limited in other articles.

We consider an MIMO downlink CR system and assume a BS transmits data to a PU and an SU. Since PU has a high priority for communication, in any channel conditions, SINR of this user must be greater than a predefined threshold based on its requirement. Also, the total transmitting power of BS is limited. By satisfying these two constraints, if SINRs of PU and SU are maximized, the best performance can be achieved. Therefore, we convert power allocation and beamforming problem to a constrained multiobjective optimization problem. The objectives, which must be maximized, are SINRs of PU and SU. The design parameters are powers that must be assigned to both users in BS and beamforming vectors of users in the transmitter (BS) and receivers.

In order to solve multiobjective optimization problems, very various methods have been proposed.<sup>29</sup> All of these solutions can be classified in two groups. In one group, which is a classic method, multiobjective optimization problem is scalarized to a single-objective optimization problem. A famous technique of this group is linear weighted sum method, in which all objectives are summed with linear weighting and the resulted objective is optimized.<sup>25,29</sup> This method cannot be applied in this paper, straightforwardly. It is because two objectives in this paper are very tangled. Also, the number of design parameters are too many. Therefore, the resulted single objective problem is not convex or semiconvex and cannot be solved, easily.

In another group of multiobjective optimization solution, nature-inspired metaheuristic algorithms are used, which obtain an approximation of optimal solution.<sup>29</sup> These methods are very sensitive to dimension of problem. In this paper, dimension of design parameters can be changed, very much. Therefore, metaheuristic methods fail to achieve a good answer in the proposed multiobjective optimization problem.

As a result in this paper, we use new methods to solve our problem. Based on the mentioned multiobjective problem, three iterative algorithms are proposed. In all algorithms at each iteration, the receiver beamforming vectors of PU and SU are extracted by assuming to know the allocated powers and BS beamforming vectors. Also, the power coefficients of two users are computed such that to ensure the power constraint of the optimization problem is satisfied at each iteration.

Difference between the proposed algorithms is in the BS beamforming design. At the first algorithm, the BS beamforming vector of each user is obtained to maximize SINR of this user, without considering to the SINR of another user and by supposing to know all other parameters. At the second algorithm, the BS beamforming vector of a user is derived at each iteration such that the SINR of both users are optimized, jointly. At the last algorithm, for computing the beamforming vector of PU, just SINR of this user is considered (the same as the first algorithm), but for SU, both SINRs of PU and SU are optimized (the same as the second algorithm). The simulation results assert that the proposed algorithms overcome the CBPA proposed in Zamiri-Jafarian and Jannat-Abad,<sup>19</sup> in various aspects of performance. Also, the computational complexity of the proposed methods are obtained.

The rest of this paper is organized as follows. In the next section, an MIMO CR system in downlink is modeled, which a BS serves to a PU and SU. In Section 3, the problem of power allocation and beamforming for the modeled system is converted to a multiobjective optimization problem, and the structure of three proposed algorithms is presented. In Section 4, computational complexity of the proposed algorithms are given. The simulation results are given in Section 5, and finally, in Section 6, this paper is concluded.

The following notations are used in the paper. Boldface capital letters denote matrices, and boldface small letters denote vectors. The superscripts  $(\cdot)^T$  and  $(\cdot)^H$  denote transpose and conjugate transpose, respectively. Also,  $E[\cdot]$  denotes statistical expectation, and  $\mathbf{I}_N$  denotes identity matrix with size of  $N \times N$ .  $\mathbf{A}(n, m)$  indicates the entry of  $n$ th row and  $m$ th column of matrix  $\mathbf{A}$ .

## 2 | SYSTEM MODEL

Assume a CR system in downlink transmission, which a BS serves to a PU and an SU, simultaneously. In Figure 1, the transceivers of the CR system are shown. As it can be seen, BS and users are equipped with multiple antennas (MIMO technique is used). The number of BS, PU, and SU antennas are  $N$ ,  $M_P$ , and  $M_S$ , respectively. In BS, the vectors of  $\mathbf{v}_P$  and  $\mathbf{v}_S$  with the length of  $N$  are utilized for beamforming of PU and SU, respectively. These two vectors are normalized such that  $\mathbf{v}_P^H \mathbf{v}_P = 1$  and  $\mathbf{v}_S^H \mathbf{v}_S = 1$ . Also,  $P_P$  and  $P_S$  are the powers assigned to PU and SU, respectively. Therefore, the transmitted signal at BS is presented by

$$\mathbf{x} = \sqrt{P_P} \mathbf{v}_P d_P + \sqrt{P_S} \mathbf{v}_S d_S, \tag{1}$$

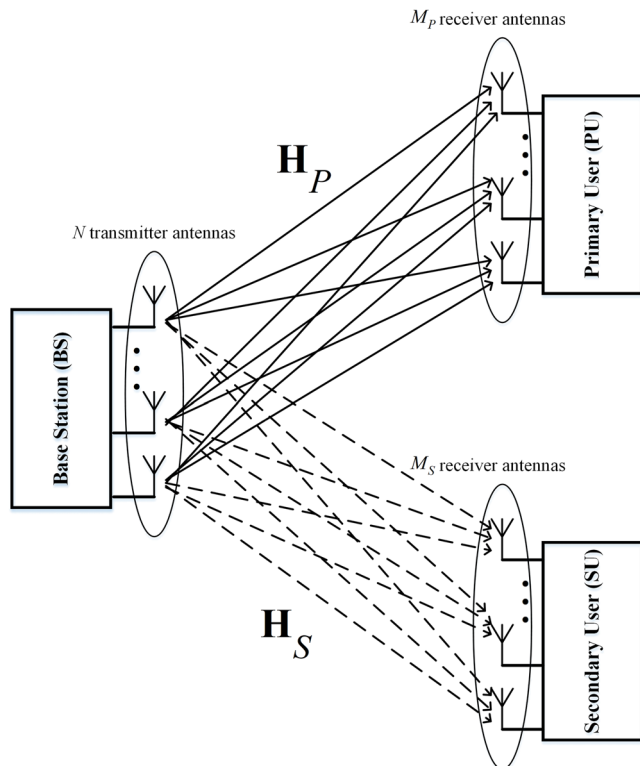


FIGURE 1 Transceiver of cognitive radio system

where the vector  $\mathbf{x}$  with the length of  $N$  is the transmitted signal from BS. Also,  $d_P$  and  $d_S$  are the data symbols of PU and SU, respectively. In here, the data symbols are generated randomly with independent and identically distributions (i.i.d.). Also, the symbol energy is normalized to one. In this figure,  $\mathbf{H}_P$  is the channel matrix with the size of  $M_P \times N$  between BS and PU. Also,  $\mathbf{H}_S$  is the channel matrix with the size of  $M_S \times N$  between BS and SU. The input signal into PU receiver antennas, which passes through the channel and adds with noise, is written as

$$\begin{aligned} \mathbf{y}_P &= \mathbf{H}_P \mathbf{x} + \mathbf{n}_P \\ &= \sqrt{P_P} \mathbf{H}_P \mathbf{v}_P d_P + \sqrt{P_S} \mathbf{H}_P \mathbf{v}_S d_S + \mathbf{n}_P, \end{aligned} \quad (2)$$

in which  $\mathbf{n}_P$  is the noise vector with the length of  $M_P$ . We suppose at each receiver antenna, white Gaussian noise with zero mean and variance of  $\sigma^2$  is added to signal. Also, the noises of all antennas are independent. Therefore,  $E[\mathbf{n}_P \mathbf{n}_P^H] = \sigma^2 \mathbf{I}_{M_P}$ . At the PU receiver side, the beamforming vector  $\mathbf{u}_P$  with the length of  $M_P$  is used to estimate the data symbols of PU. The estimated symbol at the PU receiver can be expressed as

$$\begin{aligned} \tilde{d}_P &= \mathbf{u}_P^H \mathbf{y}_P \\ &= \sqrt{P_P} \mathbf{u}_P^H \mathbf{H}_P \mathbf{v}_P d_P + \sqrt{P_S} \mathbf{u}_P^H \mathbf{H}_P \mathbf{v}_S d_S + \mathbf{u}_P^H \mathbf{n}_P. \end{aligned} \quad (3)$$

As it can be seen in (3), the estimated symbol includes three parts. The first part is the desired symbol, which must be detected. The second part is the interference induced to PU from SU, and the third part is the noise contribution over the estimated symbol. The same as PU, the estimated symbol after beamforming at the receiver of SU is

$$\tilde{d}_S = \sqrt{P_P} \mathbf{u}_S^H \mathbf{H}_S \mathbf{v}_P d_P + \sqrt{P_S} \mathbf{u}_S^H \mathbf{H}_S \mathbf{v}_S d_S + \mathbf{u}_S^H \mathbf{n}_S, \quad (4)$$

in which  $\mathbf{u}_S$  with the length of  $M_S$  is the PU receiver beamforming vector. Also,  $\mathbf{n}_S$  is the noise vector at the receiver of SU such that  $E[\mathbf{n}_S \mathbf{n}_S^H] = \sigma^2 \mathbf{I}_{M_S}$ . In the following section, we propose three algorithms in order to obtain beamforming vectors of PU and SU at the BS and receivers. Also in these algorithms, the allocated powers of PU and SU are extracted. In other words, by following the proposed iterative algorithms,  $P_P$ ,  $\mathbf{v}_P$ ,  $\mathbf{u}_P$ ,  $P_S$ ,  $\mathbf{v}_S$ , and  $\mathbf{u}_S$  are derived.

### 3 | PROPOSED POWER ALLOCATION AND BEAMFORMING ALGORITHMS

In this paper, in order to achieve an acceptable performance, the purpose is to maximize the SINRs of PU and SU. Also in CR, PU has a spectrum license and must achieve its quality of service (QoS) requirement. Therefore, we convert the power control and beamforming to a constrained multiobjective optimization problem. The objectives are the SINR of PU ( $\text{SINR}_P$ ) and SINR of SU ( $\text{SINR}_S$ ). There are two constraints, which must be satisfied. The first constraint is that  $\text{SINR}_P$  must be greater than a predefined threshold, based on the PU requirement. On the other hand, because of power limitation, the sum of PU and SU transmitted powers must be equal or less than  $P_{\max}$ , which  $P_{\max}$  is the upper power bound in BS. It means

$$P_P + P_S \leq P_{\max} \quad (5)$$

Therefore, the multiobjective optimization problem is

$$\begin{aligned} &\max_{P_P, \mathbf{u}_P, \mathbf{v}_P, P_S, \mathbf{u}_S, \mathbf{v}_S} \{ \text{SINR}_P, \text{SINR}_S \} \\ &\text{Constraints:} \\ &1) \text{SINR}_P \geq \gamma_P \\ &2) P_P + P_S \leq P_{\max} \\ &3) P_P \geq 0, \quad P_S \geq 0 \\ &4) \mathbf{v}_P^H \mathbf{v}_P = 1, \quad \mathbf{v}_S^H \mathbf{v}_S = 1. \end{aligned} \quad (6)$$

in which  $\gamma_P$  is the predefined lower threshold of PU SINR. In order to obtain  $\text{SINR}_P$  and  $\text{SINR}_S$ , we extract the desired symbol, the interference symbol, and the noise powers from (3) and (4). For PU,

$$\begin{aligned} P_D^{(P)} &= P_P \mathbf{u}_P^H \mathbf{H}_P \mathbf{v}_P \mathbf{v}_P^H \mathbf{H}_P^H \mathbf{u}_P = P_P \mathbf{v}_P^H \mathbf{H}_P^H \mathbf{u}_P \mathbf{u}_P^H \mathbf{H}_P \mathbf{v}_P \\ P_I^{(P)} &= P_S \mathbf{u}_P^H \mathbf{H}_P \mathbf{v}_S \mathbf{v}_S^H \mathbf{H}_P^H \mathbf{u}_P = P_S \mathbf{v}_S^H \mathbf{H}_P^H \mathbf{u}_P \mathbf{u}_P^H \mathbf{H}_P \mathbf{v}_S \\ P_N^{(P)} &= \sigma^2 \mathbf{u}_P^H \mathbf{u}_P, \end{aligned} \quad (7)$$

in which  $P_D^{(P)}$ ,  $P_I^{(P)}$ , and  $P_N^{(P)}$  are the desired, interference, and noise powers in (3), respectively. Thus,  $\text{SINR}_P$  is

$$\text{SINR}_P = \frac{P_D^{(P)}}{P_I^{(P)} + P_N^{(P)}} = \frac{P_P \mathbf{u}_P^H \mathbf{H}_P \mathbf{v}_P \mathbf{v}_P^H \mathbf{H}_P^H \mathbf{u}_P}{\mathbf{u}_P^H (P_S \mathbf{H}_P \mathbf{v}_S \mathbf{v}_S^H \mathbf{H}_P^H + \sigma^2 \mathbf{I}_{M_P}) \mathbf{u}_P} = \frac{P_P \mathbf{v}_P^H \mathbf{H}_P^H \mathbf{u}_P \mathbf{u}_P^H \mathbf{H}_P \mathbf{v}_P}{P_S \mathbf{v}_S^H \mathbf{H}_P^H \mathbf{u}_P \mathbf{u}_P^H \mathbf{H}_P \mathbf{v}_S + \sigma^2 \mathbf{u}_P^H \mathbf{u}_P}. \quad (8)$$

In the same way by using (4),  $\text{SINR}_S$  is

$$\text{SINR}_S = \frac{P_S \mathbf{u}_S^H \mathbf{H}_S \mathbf{v}_S \mathbf{v}_S^H \mathbf{H}_S^H \mathbf{u}_S}{\mathbf{u}_S^H (P_P \mathbf{H}_S \mathbf{v}_P \mathbf{v}_P^H \mathbf{H}_S^H + \sigma^2 \mathbf{I}_{M_S}) \mathbf{u}_S} = \frac{P_S \mathbf{v}_S^H \mathbf{H}_S^H \mathbf{u}_S \mathbf{u}_S^H \mathbf{H}_S \mathbf{v}_S}{P_P \mathbf{v}_P^H \mathbf{H}_S^H \mathbf{u}_S \mathbf{u}_S^H \mathbf{H}_S \mathbf{v}_P + \sigma^2 \mathbf{u}_S^H \mathbf{u}_S}. \quad (9)$$

In order to allocate power to users and obtain beamforming vectors, effectively, we divide problem (6) into three separate optimization models. At the first problem, we assume that the allocated powers and BS beamforming vectors for PU and SU are known, and based on the maximization of PU and SU SINRs, the receiver beamforming vectors are extracted. In the second one, power allocation is performed such that the constraints of (6) are satisfied. Finally, at the third problem, the allocated powers and receiver beamforming vectors are supposed to be known and the BS beamforming vectors for PU and SU are obtained. By solving each of three mentioned problems, two parameters are extracted based on the other parameters, and by combining the solutions, iterative algorithms are derived.

Therefore, at first,  $P_P$ ,  $\mathbf{v}_P$ ,  $P_S$ , and  $\mathbf{v}_S$  are supposed to be known and all constraints are satisfied. The purpose is to obtain  $\mathbf{u}_P$  and  $\mathbf{u}_S$ , which maximize  $\text{SINR}_P$  and  $\text{SINR}_S$ . So (6) can be replaced by

$$\max_{\mathbf{u}_P, \mathbf{u}_S} \{ \text{SINR}_P, \text{SINR}_S \}. \quad (10)$$

As it can be seen in (8),  $\text{SINR}_P$  just depends on  $\mathbf{u}_P$  and is not related to  $\mathbf{u}_S$ . Also, by considering (9), it can be derived that  $\text{SINR}_S$  does not depend on  $\mathbf{u}_P$ . Thus, the multiobjective optimization model of (10) can be represented by two single-objective optimization problems as

$$\max_{\mathbf{u}_P} \{ \text{SINR}_P \}, \max_{\mathbf{u}_S} \{ \text{SINR}_S \}. \quad (11)$$

Each of these two problems can be rewritten as a convex problem.<sup>30,31</sup> The optimized vectors of  $\mathbf{u}_P$  and  $\mathbf{u}_S$  are obtained, respectively as<sup>30,31</sup>

$$\begin{aligned} \mathbf{u}_P &= (P_S \mathbf{H}_P \mathbf{v}_S \mathbf{v}_S^H \mathbf{H}_P^H + \sigma^2 \mathbf{I}_{M_P})^{-1} \mathbf{H}_P \mathbf{v}_P \\ \mathbf{u}_S &= (P_P \mathbf{H}_S \mathbf{v}_P \mathbf{v}_P^H \mathbf{H}_S^H + \sigma^2 \mathbf{I}_{M_S})^{-1} \mathbf{H}_S \mathbf{v}_S. \end{aligned} \quad (12)$$

As it can be seen in (12), the optimized vectors of  $\mathbf{u}_P$  and  $\mathbf{u}_S$  are dependent on  $P_S$ ,  $P_P$ ,  $\mathbf{v}_S$ , and  $\mathbf{v}_P$ , which these parameters are supposed to be known in the first problem. In the second problem, by assuming to know the beamforming vectors,  $P_P$  and  $P_S$  are obtained. In order to derive power coefficients, the boundary conditions of two first constraints in (6) are considered as

- 1)  $\text{SINR}_P = \gamma_P$
- 2)  $P_P + P_S = P_{\max}$ .

In the last problem, the purpose is to extract  $\mathbf{v}_P$  and  $\mathbf{v}_S$ , which maximize  $\text{SINR}_P$  and  $\text{SINR}_S$ , by assuming that  $P_P$ ,  $\mathbf{u}_P$ ,  $P_S$ , and  $\mathbf{u}_S$  are known and all constraints are satisfied. In this case, (6) can be transformed to

$$\max_{\mathbf{v}_P, \mathbf{v}_S} \{ \text{SINR}_P, \text{SINR}_S \}. \quad (13)$$

As it can be seen in (8) and (9), each one of  $\text{SINR}_P$  and  $\text{SINR}_S$  depends on both  $\mathbf{v}_P$  and  $\mathbf{v}_S$ . Therefore, (13) cannot be considered as two separate single-objective problems. In the following, we propose three methods to obtain  $\mathbf{v}_P$  and  $\mathbf{v}_S$  based on (13).

### 3.1 | Single-objective BS beamforming method for PU and SU

In this method, the multiobjective optimization of (13) is turned into two single-objective optimization problems as

$$\max_{\mathbf{v}_P} \{ \text{SINR}_P \}, \max_{\mathbf{v}_S} \{ \text{SINR}_S \}. \quad (14)$$

Actually by applying this transformation, the dependence of  $\text{SINR}_P$  on  $\mathbf{v}_S$  is ignored. Also, we ignore that  $\text{SINR}_S$  depends on  $\mathbf{v}_P$ . In this case, the purpose of each user is to derive the beamforming vector, which maximizes its SINR, without considering the SINR of another user. As it can be seen in (8), just the numerator of  $\text{SINR}_P$  depends on  $\mathbf{v}_P$ . Thus,  $\text{SINR}_P$  is maximized in respect to  $\mathbf{v}_P$ , if the numerator of  $\text{SINR}_P$  is maximized. The same argument can be concluded for  $\text{SINR}_S$  optimization in respect to  $\mathbf{v}_S$ . The numerator of SINR is convex, and the optimized vectors in (14) are<sup>32</sup>

$$\begin{aligned} \mathbf{v}_P &= 1/\sqrt{\eta_P} \mathbf{H}_P^H \mathbf{u}_P \\ \mathbf{v}_S &= 1/\sqrt{\eta_S} \mathbf{H}_S^H \mathbf{u}_S \end{aligned}, \quad (15)$$

in which  $\eta_P = \mathbf{u}_P^H \mathbf{H}_P \mathbf{H}_P^H \mathbf{u}_P$  and  $\eta_S = \mathbf{u}_S^H \mathbf{H}_S \mathbf{H}_S^H \mathbf{u}_S$  are the normalized factors that lead to  $\mathbf{v}_P^H \mathbf{v}_P = 1$  and  $\mathbf{v}_S^H \mathbf{v}_S = 1$ .

As it was mentioned before, we divide problem (6) into three separate problems, which by solving each of them, two parameters are obtained based on the other ones. By considering these three problems, in Table 1, a new iterative algorithm is proposed for beamforming and power control, which we call it single-objective BS beamforming (SBB) algorithm.

In this table,  $P_P^{[r]}$ ,  $P_S^{[r]}$ ,  $\mathbf{v}_P^{[r]}$ ,  $\mathbf{v}_S^{[r]}$ ,  $\mathbf{u}_P^{[r]}$ , and  $\mathbf{u}_S^{[r]}$  are power of PU, power of SU, transmitter beamforming vector of PU, transmitter beamforming vector of SU, receiver beamforming vector of PU, and receiver beamforming vector of SU, which are derived at  $r$ th iteration of algorithm, respectively. Also,  $0 \leq \alpha \leq 1$  is a regulator coefficient that is determined at the initialization step.

As it can be seen in Table 1, at each iteration after obtaining power coefficients of PU and SU, if these coefficient values are not negative, the PU and SU beamforming vectors in BS and receivers are computed based on (12) and (15). But, if a negative value is obtained for one of them, then we put  $P_P = P_{\max}$  and  $P_S = 0$ . In other words, just data communication between BS and PU is permitted, and no service is given to SU. It is because when the channel fading is very severe, it is not possible to keep  $\text{SINR}_P$  greater than  $\gamma_P$  and at the same time, BS serves SU. In this case, the total power is allocated to PU. Therefore, there is no interference induced to PU, and by substituting  $P_S = 0$  into  $\mathbf{u}_P$  in (12), we have  $\mathbf{u}_P = \mathbf{H}_P \mathbf{v}_P$ . After replacing  $P_P = P_{\max}$  and  $P_S = 0$  in the proposed algorithm, the main iteration is broken off, and a new iteration begun, which  $\mathbf{u}_P$  and  $\mathbf{v}_P$  are obtained from it.

### 3.2 | Multiobjective BS beamforming method for PU and SU

In this method, in order to obtain a solution for (13), at first, we assume that  $\mathbf{v}_S$  is known and therefore (13) can be represented as

$$\max_{\mathbf{v}_P} \{ \text{SINR}_P, \text{SINR}_S \}. \quad (16)$$

We already know that just the numerator of  $\text{SINR}_P$  depends on  $\mathbf{v}_P$ . On the other hand, by considering (9), it can be seen that just the denominator of  $\text{SINR}_S$  depends on  $\mathbf{v}_P$ . Therefore, the multiobjective optimization of (16) is equivalent to maximization of numerator of  $\text{SINR}_P$  and minimization of denominator of  $\text{SINR}_S$  in respect to  $\mathbf{v}_P$ . Thus, (16) is transformed to

$$\max_{\mathbf{v}_P} \{ \mathbf{v}_P^H \mathbf{H}_P^H \mathbf{u}_P \mathbf{u}_P^H \mathbf{H}_P \mathbf{v}_P \}, \min_{\mathbf{v}_P} \{ \mathbf{v}_P^H \mathbf{H}_S^H \mathbf{u}_S \mathbf{u}_S^H \mathbf{H}_S \mathbf{v}_P \}. \quad (17)$$



**TABLE 1** The first proposed algorithm: SBB

<p><b>Initialize <math>P_P, P_S, \mathbf{v}_P,</math> and <math>\mathbf{v}_S</math></b></p> <p><math>r = 0</math></p> <p><math>P_P^{[r]} = \alpha P_{\max}</math> and <math>P_S^{[r]} = (1 - \alpha) P_{\max}</math></p> <p><math>\mathbf{v}_P^{[r]} = [1 \ 0 \ \dots \ 0]^T</math> and <math>\mathbf{v}_S^{[r]} = [1 \ 0 \ \dots \ 0]^T</math></p> <p><b>Calculate <math>\mathbf{u}_P^{[r]}, \mathbf{u}_S^{[r]}, a_D^{[r]}, b_I^{[r]},</math> and <math>c_N^{[r]}</math> as</b></p> <p><math>\mathbf{u}_P^{[r]} = \left( P_S^{[r]} \mathbf{H}_P \mathbf{v}_S^{[r]} \mathbf{v}_S^{[r]H} \mathbf{H}_P^H + \sigma^2 \mathbf{I}_{M_P} \right)^{-1} \mathbf{H}_P \mathbf{v}_P^{[r]}</math></p> <p><math>\mathbf{u}_S^{[r]} = \left( P_P^{[r]} \mathbf{H}_S \mathbf{v}_P^{[r]} \mathbf{v}_P^{[r]H} \mathbf{H}_S^H + \sigma^2 \mathbf{I}_{M_S} \right)^{-1} \mathbf{H}_S \mathbf{v}_S^{[r]}</math></p> <p><math>a_D^{[r]} = \mathbf{u}_P^{[r]H} \mathbf{H}_P \mathbf{v}_P^{[r]} \mathbf{v}_P^{[r]H} \mathbf{H}_P^H \mathbf{u}_P^{[r]}</math></p> <p><math>b_I^{[r]} = \mathbf{u}_P^{[r]H} \mathbf{H}_P \mathbf{v}_S^{[r]} \mathbf{v}_S^{[r]H} \mathbf{H}_P^H \mathbf{u}_P^{[r]}</math></p> <p><math>c_N^{[r]} = \sigma^2 \mathbf{u}_P^{[r]H} \mathbf{u}_P^{[r]}</math></p>
<p>Iterations:</p> <p><math>r = r + 1</math></p> <p>Obtain <math>P_P^{[r]}</math> and <math>P_S^{[r]}</math> from the following equations:</p> $\frac{P_P^{[r]} a_D^{[r-1]}}{P_P^{[r]} a_D^{[r-1]} + c_N^{[r-1]}} = \gamma_P$ $P_P^{[r]} + P_S^{[r]} = 1$ <p>if <math>P_P^{[r]} \geq 0</math> and <math>P_S^{[r]} \geq 0</math>, then</p> $\mathbf{v}_P^{[r]} = \mathbf{H}_P^H \mathbf{u}_P^{[r-1]}, \mathbf{v}_P^{[r]} = \frac{1}{\sqrt{\mathbf{v}_P^{[r]H} \mathbf{v}_P^{[r]}}} \mathbf{v}_P^{[r]}$ $\mathbf{v}_S^{[r]} = \mathbf{H}_S^H \mathbf{u}_S^{[r-1]}, \mathbf{v}_S^{[r]} = \frac{1}{\sqrt{\mathbf{v}_S^{[r]H} \mathbf{v}_S^{[r]}}} \mathbf{v}_S^{[r]}$ <p>Calculate <math>\mathbf{u}_P^{[r]}, \mathbf{u}_S^{[r]}, a_D^{[r]}, b_I^{[r]},</math> and <math>c_N^{[r]}</math> the same as the initialization step</p> <p>else:</p> <p>Break iteration <math>r</math></p> <p><math>P_P = P_{\max}</math> and <math>P_S = 0</math></p> <p><math>i = 0</math></p> $\mathbf{v}_P^{[i]} = \mathbf{H}_P^H \mathbf{u}_P^{[r]}, \quad \mathbf{v}_P^{[i]} = \frac{1}{\sqrt{\mathbf{v}_P^{[i]H} \mathbf{v}_P^{[i]}}} \mathbf{v}_P^{[i]}$ <p><math>\mathbf{u}_P^{[i]} = \mathbf{H}_P \mathbf{v}_P^{[i]}</math></p> <p>iterations:</p> <p><math>i = i + 1;</math></p> $\mathbf{v}_P^{[i]} = \mathbf{H}_P^H \mathbf{u}_P^{[i-1]}, \mathbf{v}_P^{[i]} = \frac{1}{\sqrt{\mathbf{v}_P^{[i]H} \mathbf{v}_P^{[i]}}} \mathbf{v}_P^{[i]}$ <p><math>\mathbf{u}_P^{[i]} = \mathbf{H}_P \mathbf{v}_P^{[i]}</math></p>

It must be noted that  $\mathbf{H}_S^H \mathbf{u}_S \mathbf{u}_S^H \mathbf{H}_S$  is a Hermitian square matrix with the size of  $N \times N$ . Therefore, by using singular value decomposition (SVD) technique, this matrix can be represented as<sup>33</sup>

$$\mathbf{G}_S \Delta_S \mathbf{G}_S^H = \mathbf{H}_S^H \mathbf{u}_S \mathbf{u}_S^H \mathbf{H}_S, \quad (18)$$

where  $\Delta_S$  is a diagonal matrix, which its main diagonal consists all  $N$  singular values of  $\mathbf{H}_S^H \mathbf{u}_S \mathbf{u}_S^H \mathbf{H}_S$  and it is assumed that these singular values are in descending order ( $\Delta_S(n, n) \geq \Delta_S(n + 1, n + 1)$ ). Also,  $\mathbf{G}_S = [\mathbf{g}_S^{(1)} \ \dots \ \mathbf{g}_S^{(N)}]$  is the singular matrix that contains all singular vectors of  $\mathbf{H}_S^H \mathbf{u}_S \mathbf{u}_S^H \mathbf{H}_S$ .  $\mathbf{g}_S^{(n)}$  is the singular vector corresponding to  $n$ th singular value. On the other hand,  $\mathbf{H}_S^H \mathbf{u}_S \mathbf{u}_S^H \mathbf{H}_S$  is an outer product of a vector ( $\mathbf{H}_S^H \mathbf{u}_S$ ) and its Hermitian. Therefore, the rank of this matrix is one and just has a nonzero singular value.<sup>34</sup>  $N - 1$  other singular-values of this matrix are zero.

If  $\mathbf{v}_P$  is equal to one of the singular vectors corresponding to zero singular values of  $\mathbf{H}_S^H \mathbf{u}_S \mathbf{u}_S^H \mathbf{H}_S$ , then  $\mathbf{v}_P^H \mathbf{H}_S^H \mathbf{u}_S \mathbf{u}_S^H \mathbf{H}_S \mathbf{v}_P = 0$ . Therefore, we put  $\mathbf{v}_P$  as a linear combination of singular vectors corresponding to zero singular values of  $\mathbf{H}_S^H \mathbf{u}_S \mathbf{u}_S^H \mathbf{H}_S$  that cause to minimize  $\mathbf{v}_P^H \mathbf{H}_S^H \mathbf{u}_S \mathbf{u}_S^H \mathbf{H}_S \mathbf{v}_P$  in (17):

$$\mathbf{v}_P = \sum_{k=2}^N \lambda_S^{(k)} \mathbf{g}_S^{(k)}, \quad (19)$$

where  $\lambda_S^{(k)}$  is the weighting coefficient of  $k$ th singular vector of  $\mathbf{H}_S^H \mathbf{u}_S \mathbf{u}_S^H \mathbf{H}_S$  (which corresponding to a zero singular value for  $2 \leq k \leq N$ ). Equation (19) can be rewritten in vector form as

$$\mathbf{v}_P = \bar{\mathbf{G}}_S \lambda_S, \quad (20)$$

in which  $\bar{\mathbf{G}}_S = [\mathbf{g}_S^{(2)} \dots \mathbf{g}_S^{(N)}]$  is a matrix with the size of  $N \times (N - 1)$  that includes all singular vectors corresponding to all zero singular values of  $\mathbf{H}_S^H \mathbf{u}_S \mathbf{u}_S^H \mathbf{H}_S$ . Also,  $\lambda_S = [\lambda_S^{(2)} \dots \lambda_S^{(N)}]^T$  is a vector with the length of  $(N - 1)$ . As we mentioned before, by satisfying (20), the minimization problem in (17) is solved. Just the vector  $\lambda_S$  must be developed such that  $\mathbf{v}_P^H \mathbf{H}_P^H \mathbf{u}_P \mathbf{u}_P^H \mathbf{H}_P \mathbf{v}_P$  in (17) is maximized. Therefore, by replacing (20) in (17), this optimization problem is transformed to

$$\max_{\lambda_S} \{ \lambda_S^H \bar{\mathbf{G}}_S^H \mathbf{H}_P^H \mathbf{u}_P \mathbf{u}_P^H \mathbf{H}_P \bar{\mathbf{G}}_S \lambda_S \}. \quad (21)$$

The same as (14), (21) is a convex problem that the solution of it is  $\lambda_S = \bar{\mathbf{G}}_S^H \mathbf{H}_P^H \mathbf{u}_P$ ,<sup>30</sup> which by replacing in (20),

$$\mathbf{v}_P = \bar{\mathbf{G}}_S \bar{\mathbf{G}}_S^H \mathbf{H}_P^H \mathbf{u}_P. \quad (22)$$

In the same way, in order to obtain  $\mathbf{v}_S$ , we assume  $\mathbf{v}_P$  is known. Thus, (13) is replaced by

$$\max_{\mathbf{v}_S} \{ \text{SINR}_P, \text{SINR}_S \}. \quad (23)$$

**TABLE 2** The second proposed algorithm: MBB

<p><b>Initialize <math>P_P, P_S, \mathbf{v}_P</math>, and <math>\mathbf{v}_S</math></b>  <math>r = 0</math>  <math>P_P^{[r]} = \alpha P_{\max}</math> and <math>P_S^{[r]} = (1 - \alpha) P_{\max}</math>  <math>\mathbf{v}_P^{[r]} = [10 \dots 0]^T</math> and <math>\mathbf{v}_S^{[r]} = [10 \dots 0]^T</math>  <b>Calculate <math>\mathbf{u}_P^{[r]}, \mathbf{u}_S^{[r]}, a_D^{[r]}, b_I^{[r]}</math>, and <math>c_N^{[r]}</math> the same as the initialization step of SBB</b></p>
<p>Iterations:  <math>r = r + 1</math>            Obtain <math>P_P^{[r]}</math> and <math>P_S^{[r]}</math> from the following equations:  <math display="block">\frac{P_P^{[r]} a_D^{[r-1]}}{P_S^{[r]} b_I^{[r-1]} + c_N^{[r-1]}} = \gamma_P</math> <math display="block">P_P^{[r]} + P_S^{[r]} = 1</math></p>
<p>if <math>P_P^{[r]} \geq 0</math> and <math>P_S^{[r]} \geq 0</math>, then            Compute singular vectors of <math>\mathbf{A}_S^{[r]} = \mathbf{H}_S^H \mathbf{u}_S^{[r-1]} \mathbf{u}_S^{[r-1]H} \mathbf{H}_S</math> and save singular vectors corresponding to all zero singular values of <math>\mathbf{A}_S^{[r]}</math> in matrix of <math>\bar{\mathbf{G}}_S^{[r]}</math>  <math display="block">\mathbf{v}_P^{[r]} = \bar{\mathbf{G}}_S^{[r]} \bar{\mathbf{G}}_S^{[r]H} \mathbf{H}_P^H \mathbf{u}_P^{[r-1]}, \mathbf{v}_P^{[r]} = \frac{1}{\sqrt{\mathbf{v}_P^{[r]H} \mathbf{v}_P^{[r]}}} \mathbf{v}_P^{[r]}</math>           Compute singular vectors of <math>\mathbf{A}_P^{[r]} = \mathbf{H}_P^H \mathbf{u}_P^{[r-1]} \mathbf{u}_P^{[r-1]H} \mathbf{H}_P</math> and save singular vectors corresponding to all zero singular values of <math>\mathbf{A}_P^{[r]}</math> in matrix of <math>\bar{\mathbf{G}}_P^{[r]}</math>  <math display="block">\mathbf{v}_S^{[r]} = \bar{\mathbf{G}}_P^{[r]} \bar{\mathbf{G}}_P^{[r]H} \mathbf{H}_S^H \mathbf{u}_S^{[r-1]}, \mathbf{v}_S^{[r]} = \frac{1}{\sqrt{\mathbf{v}_S^{[r]H} \mathbf{v}_S^{[r]}}} \mathbf{v}_S^{[r]}</math>           Calculate <math>\mathbf{u}_P^{[r]}, \mathbf{u}_S^{[r]}, a_D^{[r]}, b_I^{[r]}</math> and <math>c_N^{[r]}</math> the same as the initialization step</p>
<p>else:            Break iteration <math>r</math>  <math>P_P = P_{\max}</math> and <math>P_S = 0</math>  <math>i = 0</math>  <math display="block">\mathbf{v}_P^{[i]} = \mathbf{H}_P^H \mathbf{u}_P^{[r]}, \mathbf{v}_P^{[i]} = \frac{1}{\sqrt{\mathbf{v}_P^{[i]H} \mathbf{v}_P^{[i]}}} \mathbf{v}_P^{[i]}</math> <math display="block">\mathbf{u}_P^{[i]} = \mathbf{H}_P \mathbf{v}_P^{[i]}</math>           iterations:  <math>i = i + 1</math>;  <math display="block">\mathbf{v}_P^{[i]} = \mathbf{H}_P^H \mathbf{u}_P^{[i-1]}, \mathbf{v}_P^{[i]} = \frac{1}{\sqrt{\mathbf{v}_P^{[i]H} \mathbf{v}_P^{[i]}}} \mathbf{v}_P^{[i]}</math> <math display="block">\mathbf{u}_P^{[i]} = \mathbf{H}_P \mathbf{v}_P^{[i]}</math></p>



The similar procedure to extract  $\mathbf{v}_P$  is followed to obtain  $\mathbf{v}_S$  as

$$\mathbf{v}_S = \bar{\mathbf{G}}_P \bar{\mathbf{G}}_P^H \mathbf{H}_S^H \mathbf{u}_S, \quad (24)$$

in which  $\bar{\mathbf{G}}_P = [\mathbf{g}_P^{(2)} \dots \mathbf{g}_P^{(N)}]$  is a matrix with the size of  $N \times (N - 1)$  that includes all singular vectors corresponding to all zero singular values of  $\mathbf{H}_P^H \mathbf{u}_P \mathbf{u}_P^H \mathbf{H}_P$ .

Based on the proposed method here, we produce another algorithm, which is called multiobjective BS beamforming (MBB) algorithm. The procedure of MBB is presented in Table 2.

Similar to SBB in MBB algorithm, if  $P_P^{[r]}$  or  $P_S^{[r]}$  is negative, then the main iteration is broken of and  $P_P$  and  $P_S$  are assigned to  $P_{max}$  and zero, respectively. After that, by applying a new iterative algorithm,  $\mathbf{v}_P$  and  $\mathbf{v}_S$  are computed.

### 3.3 | Single-objective for PU, multiobjective for SU, BS beamforming method

In this method, the BS beamforming vector of PU is obtained such that  $\text{SINR}_P$  is maximized, without considering to  $\text{SINR}_S$  improvement. Thus,  $\mathbf{v}_P$  is computed based on (15). But  $\mathbf{v}_S$  is developed such that maximizes both SINRs the same as (23). Therefore,  $\mathbf{v}_S$  is extracted similar to (24). In other words in this case, since PU has the spectrum access license, SU helps PU to improve its performance. But PU just tries to maximize its SINR. Based on this strategy, the third proposed algorithm is derived, which at each iteration,  $\mathbf{v}_P^{[r]}$  is computed from SBB in Table 1 and  $\mathbf{v}_S^{[r]}$  is computed from MBB in Table 2. All other steps of the proposed algorithm are the same as SBB and MBB. We call this algorithm PU single-objective, SU multiobjective BS beamforming (PSSMBB).

## 4 | COMPUTATIONAL COMPLEXITY

In this section, the computational complexity order of the proposed algorithms is obtained. The most complexity in matrix algebra belongs to calculation of matrix SVD, matrix inversion, and matrix multiplication. SVD and inversion of a  $n \times n$  matrix are done in  $O(n^3)$  and  $O(n^{2.373})$ , respectively.<sup>35</sup> Also, the complexity order of two matrix multiplication with the size  $n \times m$  and  $m \times p$  is  $O(nmp)$ .<sup>36</sup>

We derive the computational complexity of each algorithm at  $r$ th iteration. Firstly, SBB algorithm, presented in Table 1, is considered. In order to calculate  $\mathbf{v}_P^{[r]}$ , a  $N \times M_P$  matrix is multiplied to a vector with the length  $M_P$ , which is done in  $O(NM_P)$ . Similarly, calculation of  $\mathbf{v}_S^{[r]}$  requires a complexity of  $O(NM_S)$ . For calculation of  $\mathbf{u}_P^{[r]}$  and  $\mathbf{u}_S^{[r]}$ , matrix multiplication and inversion are needed, which can be implemented by complexity of  $O(\max\{NM_P, M_P^{2.373}\})$  and  $O(\max\{NM_S, M_S^{2.373}\})$ , respectively. Therefore, the complexity order of SBB algorithm is  $O(\max\{NM_P, M_P^{2.373}, NM_S, M_S^{2.373}\})$ . By assuming  $N = M_P = M_S$ ,  $r$ th iteration of SBB algorithm is implemented by complexity of  $O(N^{2.373})$ .

In order to calculate BS beamformers ( $\mathbf{v}_P^{[r]}$  and  $\mathbf{v}_S^{[r]}$ ) in MBB algorithm, presented in Table 2, firstly,  $\mathbf{A}_S^{[r]}$  and  $\mathbf{A}_P^{[r]}$  must be obtained, which this procedures require complexity of  $O(\max\{NM_S, N^2\})$  and  $O(\max\{NM_P, N^2\})$ , respectively. In the next step, SVD of these two matrices must be calculated, which both of them involves complexity of  $O(4N^3)$ . Thus, computational complexity order of SVD procedure is  $O(\max\{8N^3, NM_P, NM_S\})$ . For calculation of  $\mathbf{v}_P^{[r]}$ , firstly, the matrix  $\bar{\mathbf{G}}_S^{[r]}$  with the size  $N \times N - 1$  must be multiplied to its Hermitian that requires complexity of  $O(N^3)$ . Then, the resulted matrix is multiplied to the vector of  $\mathbf{H}_P^H \mathbf{u}_P^{[r-1]}$  that is implemented by complexity of  $O(N^2)$ . Also, calculation of  $\mathbf{H}_P^H \mathbf{u}_P^{[r-1]}$  needs complexity of  $O(NM_P)$ . As a result, the computational complexity order of  $\mathbf{v}_P^{[r]}$  is  $O(\max\{9N^3, NM_P, NM_S\})$ , which is the same as complexity of  $\mathbf{v}_S^{[r]}$  calculation. Since the calculation of beamforming vectors of users is the same as SBB algorithm, this step requires complexity of  $O(\max\{NM_P, M_P^{2.373}, NM_S, M_S^{2.373}\})$ . Thus, it is concluded that the computational complexity order of  $r$ th iteration at MBB algorithm is  $O(\max\{9N^3, NM_P, NM_S, M_S^{2.373}, M_P^{2.373}\})$ , which by assuming  $N = M_P = M_S$  becomes  $O(9N^3)$ .

Since PSSMBB algorithm is combination of SBB and MBB algorithms, straightforwardly, it is resulted that the computational complexity order of this algorithm is  $O(\max\{5N^3, NM_P, NM_S, M_S^{2.373}, M_P^{2.373}\})$ , which by assuming  $N = M_P = M_S$  becomes  $O(5N^3)$ . It is obvious that SBB and MBB algorithms have the minimum and maximum complexity, respectively. In the following, the performance of three proposed algorithms are evaluated in computer simulations.

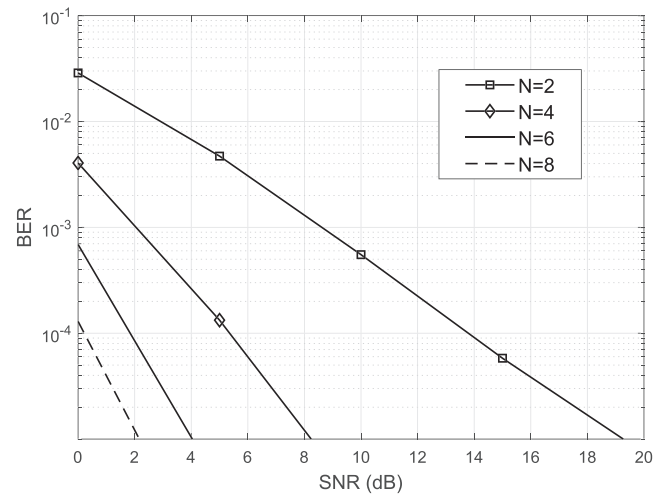
## 5 | SIMULATION RESULTS

In this section, our proposed methods are compared with CBPA algorithm presented in Zamiri-Jafarian Hossein and Jannat-Abad<sup>19</sup> in computer simulations. The reason that we choose CBPA algorithm as a benchmark is that the CR system model illustrated in Zamiri-Jafarian Hossein and Jannat-Abad<sup>19</sup> is exactly the same as the presented model in here. Actually, very various beamforming or power allocation algorithms have been proposed in articles. But in almost of these papers, the CR system models are different from our model.

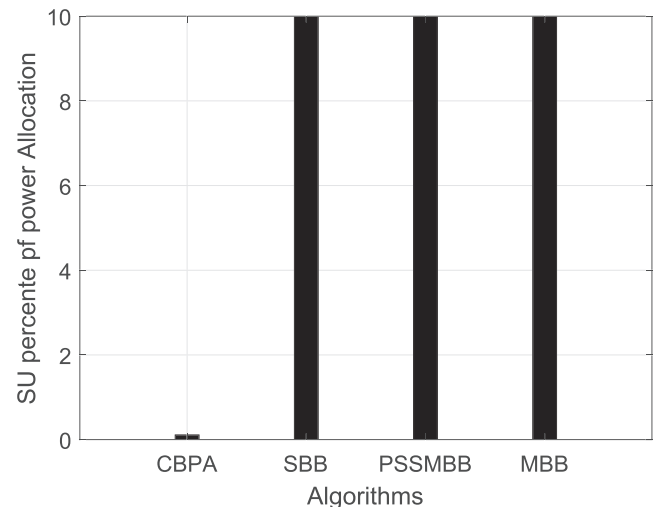
In all algorithms, let  $\alpha = 0.9$  and  $\gamma_p = 20\text{dB}$ . Also, 20 iterations for each algorithm is considered. QPSK modulation is used at BS. The simulation results are averaged over  $10^5$  different channels. The elements of channel impulse response are independent and have zero mean Gaussian distributions with normalized variance.

In Figure 2, bit error rate (BER) of PU versus signal to noise ratio (SNR) is shown for different number of BS antennas ( $N$ ), when the number of PU and SU antennas are 2 ( $M_p = M_s = 2$ ). For given  $M_p$  and  $M_s$ , the BER of PU in all algorithms are the same. Thus, in Figure 2, just a single curve is drawn for all four algorithms. These results because of that, the priority of system for transmission is PU. In other words in all methods, beamforming and power allocation are done such that at least  $\text{SINR}_p = \gamma_p$ , or if this constraint cannot be satisfied, no power is allocated to SU. Therefore, PU achieves the best performance, which is the same in all algorithms. As it can be seen in Figure 2, with increasing the number of BS antennas, the BER of PU is reduced.

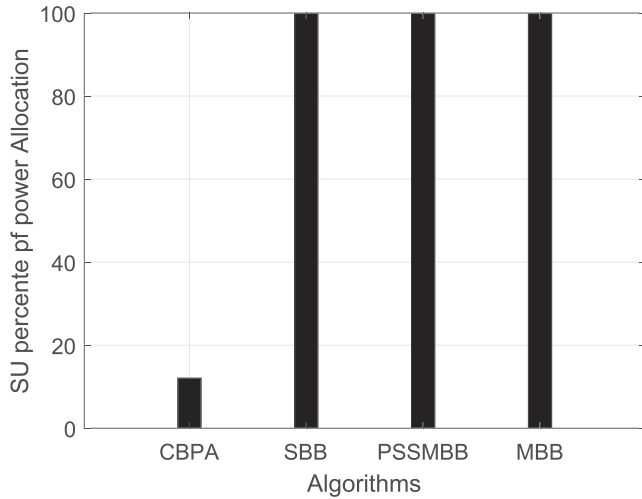
The difference between the algorithms appears in the SU performance. In Figure 3, the percent of time that power can be allocated to SU ( $P_s \neq 0$ ) is shown when  $N = 4$ ,  $M_p = M_s = 2$  and  $\text{SNR} = 15\text{dB}$ . As it can be seen, whereas the times that power is allocated to SU for CBPA is much less than 1%, but for proposed algorithms, are greater than 10%. This percentage for proposed algorithms is approximately around 80% (The figure just shows the percentage lower than 10



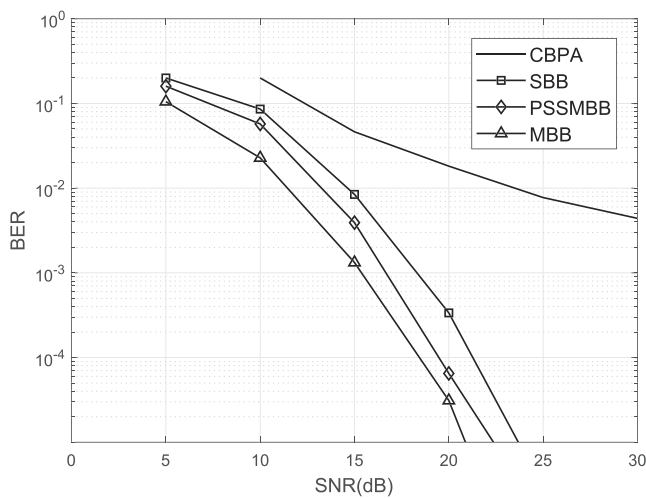
**FIGURE 2** Bit error rate (BER) of primary user (PU) versus signal to noise ratio (SNR) for different number of base station (BS) antennas ( $N$ ) when  $M_p = M_s = 2$



**FIGURE 3** Percentage of power allocation to secondary user (SU) when  $N = 4$ ,  $M_p = M_s = 2$ , and signal to noise ratio (SNR) = 15dB



**FIGURE 4** Percentage of power allocation to secondary user (SU) when  $N = 4$ ,  $M_P = M_S = 2$  and signal to noise ratio (SNR) = 25dB



**FIGURE 5** Bit error rate (BER) of secondary user (SU) (which can achieve service from base station [BS]) versus signal to noise ratio [SNR] when  $N = 4$  and  $M_P = M_S = 2$

because of that the performance of CBPA can be seen). Therefore, the proposed methods strongly overcome CBPA. The same results are presented in Figure 4, where  $SNR = 25dB$ . It can be concluded that by using the proposed algorithms, in different channel conditions, the chance of power allocation to SU is tremendously higher than CBPA in both low and high SNRs.

In Figure 5, the BER of SU (that can achieve communication service from BS) versus SNR is shown when  $N = 4$ ,  $M_P = M_S = 2$ . The BERs of proposed algorithms are much less than CBPA. Also, it can be seen that the MBB algorithm achieves the best performance between all of them. It is because in MBB, each user, in addition to try to improve its SINR, considers the SINR of another user, which leads to improving the performance of SU and PU, simultaneously. After MBB, PSSMBB achieves lower BER, and between three proposed algorithms, SBB achieves the worst performance. In SBB, each user just wants to develop the transmitter beamforming vector that maximizes its SINR.

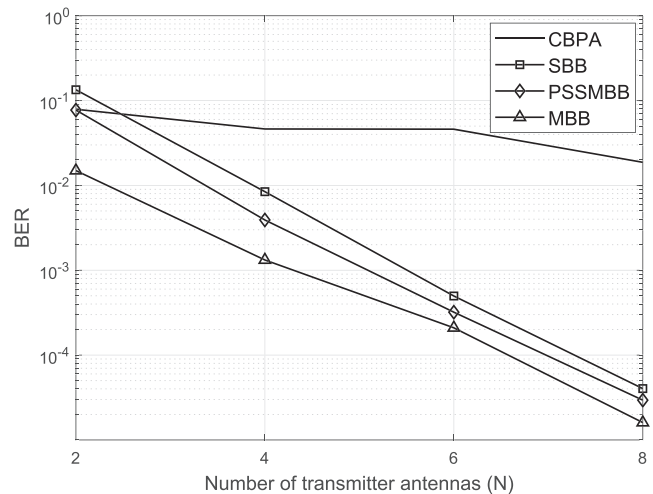
Of course, achieving the best performance for MBB is with the cost of higher computational complexity. As it can be seen in Table 2, at each iteration of MBB algorithm, SVD of two matrices must be calculated. But in SBB, there is no need to calculate SVD. Also in PSSMBB, one SVD calculation of matrix is needed, which cause to more complexity than SBB and less complexity than MBB.

In addition, the results in Figure 5 show that when CBPA is applied, for  $SNR < 10dB$ , no service is given to SU, while by using the proposed algorithms, just for  $SNR = 0dB$ , service is not assigned to SU.

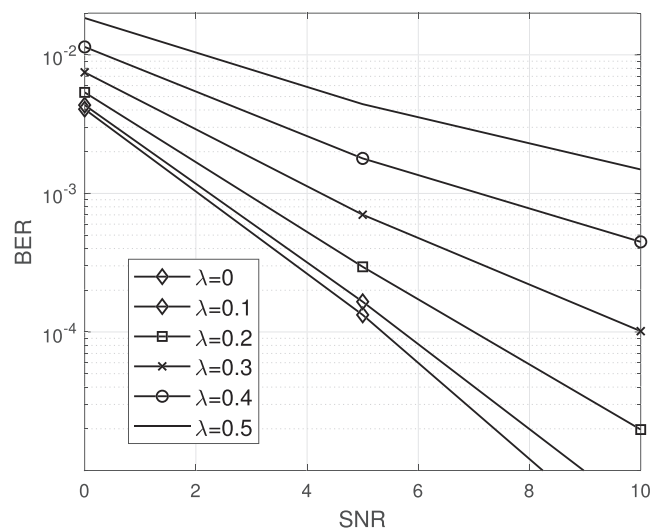
In Figure 6, BER versus number of BS antennas ( $N$ ) is shown when  $M_P = M_S = 2$  and  $SNR = 15dB$ . In this figure, by increasing  $N$ , the BER of proposed methods are reduced, while the BER of CBPA is approximately constant. These results assert the superiority of proposed algorithms in comparison with CBPA.

In the following simulations, we assume the transceivers cannot estimate the channel state information (CSI), perfectly. In order to show the effect of imperfect CSI at the transmitter and receiver sides, it is supposed that the channel matrices

**FIGURE 6** Bit error rate (BER) of bit error rate (SU) versus number of base station (BS) antennas ( $N$ ) when  $N = 4$ ,  $M_p = M_s = 2$  and signal to noise ratio (SNR) = 15dB



**FIGURE 7** Bit error rate (BER) of primary user (PU) versus signal to noise ratio (SNR) in multiobjective base station beamforming (MBB) algorithm for imperfect channel state information (CSI) when  $N = 4$  and  $M_p = M_s = 2$



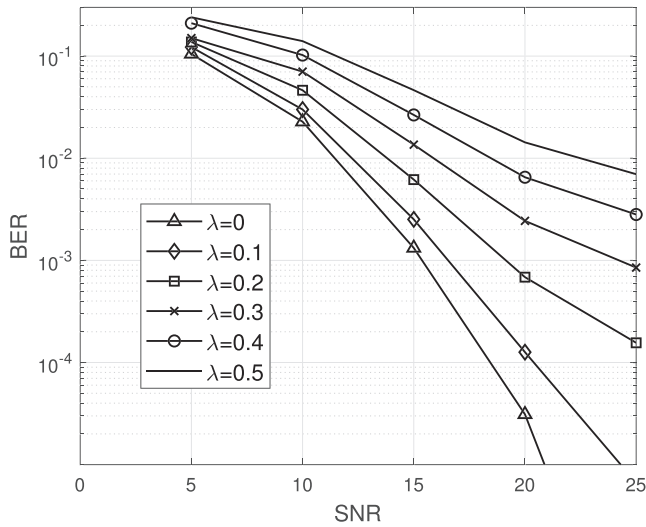
include two parts as

$$\begin{aligned} \mathbf{H}_P &= \beta \mathbf{H}_P^E + \lambda \mathbf{H}_P^\Delta \\ \mathbf{H}_S &= \beta \mathbf{H}_S^E + \lambda \mathbf{H}_S^\Delta \end{aligned} \quad (25)$$

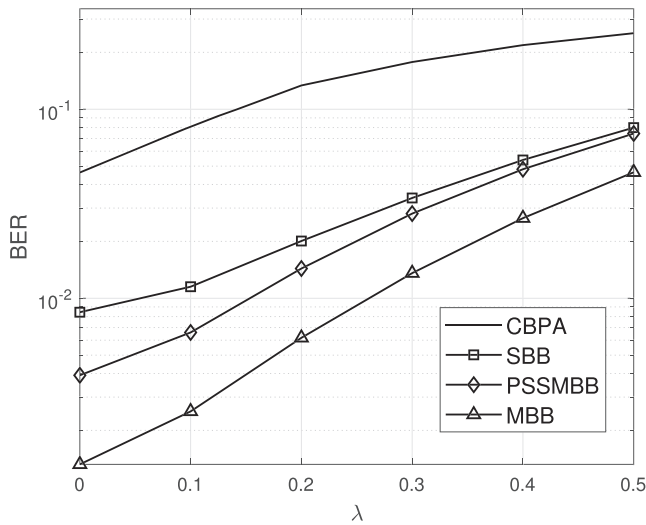
in which  $\mathbf{H}_P^E$  and  $\mathbf{H}_S^E$  are the parts of channel matrices that are correctly estimated and are used in calculation of beamforming vectors. Also,  $\mathbf{H}_P^\Delta$  and  $\mathbf{H}_S^\Delta$  are the parts of channel matrices, which cannot be estimated.  $\beta$  and  $\lambda$  are the weights of estimated and unestimated channel matrices, respectively, such that  $\sqrt{\beta^2 + \lambda^2} = 1$ . All entries of  $\mathbf{H}_P^E$ ,  $\mathbf{H}_S^E$ ,  $\mathbf{H}_P^\Delta$ , and  $\mathbf{H}_S^\Delta$  are generated randomly with Gaussian distribution, zero mean and variance of one. Thus, by increasing  $\lambda$ , the power contribution of unestimated parts are increased while the power of channel matrices are fixed.

In Figures 7 and 8, BER of PU and SU versus SNR in MBB algorithm, respectively, are shown for different values of  $\lambda$  when  $N = 4$  and  $M_p = M_s = 2$ . For both PU and SU, by increasing the weight of unestimated part of channel, BER performance is reduced, which it is clear. But, even for large values of  $\lambda$ , by SNR increasing, BER is reduced. These results confirm that MBB algorithm is robust against imperfect CSI. Similar results are obtained for SBB and PSSMBB algorithms.

In order to compare BER performance of all algorithms in condition of imperfect CSI, clearly, in Figure 9, BER of SU versus  $\lambda$  is shown for the proposed and CBPA algorithms, in which  $N = 4$ ,  $M_p = M_s = 2$  and SNR = 15dB. This figure assert that the proposed algorithms overcome CBPA when the channel estimation is not perfect. Also, in Figure 9, it is seen that by increasing  $\lambda$ , BER of PSSMBB algorithm increases faster than two other proposed algorithms. These results show that the robustness of PSSMBB against imperfect CSI is lower than SBB and MBB algorithms. It is because PSSMBB is a combination of SBB and MBB.



**FIGURE 8** Bit error rate (BER) of secondary user (SU) versus signal to noise ratio (SNR) in multiobjective base station beamforming (MBB) algorithm for imperfect channel state information (CSI) when  $N = 4$  and  $M_p = M_s = 2$



**FIGURE 9** Bit error rate (BER) of secondary user (SU) versus  $\lambda$  when  $N = 4$ ,  $M_p = M_s = 2$  and signal to noise ratio (SNR) = 15 dB


## 6 | CONCLUSIONS

In this paper, we proposed three new iterative algorithms (SBB, PSSMBB, and MBB) for power allocation and beamforming in a downlink CR communication system, which a BS serves to a PU and an SU. The proposed algorithms are derived based on a multiobjective optimization problem. The purpose of this problem is to maximize the SINRs of PU and SU by considering two constraints. The first constraint is that the SINR of PU must be equal or greater than a predefined threshold, in order to ensure that PU achieves its demand requirement. The second one is that the total power in BS is limited. At each iteration, receiver beamforming vectors (for SU and PU) are obtained the same for all algorithms by maximizing SINRs of both users and assuming that allocated powers and BS beamforming vectors are known. In order to derive the BS beamforming vectors for PU and SU in SBB algorithm, each user optimize its SINR without considering the other user achievement. In MBB algorithm, both users maximize their SINRs and SINR of another user. In PSSMBB algorithm, beamforming vectors of PU and SU are obtained the same as SBB and MBB, respectively. The computational complexity order of proposed algorithms was obtained. Simulation results show extremely superiority of the proposed algorithms in performance comparison with CBPA algorithm presented in Zamiri-Jafarian Hossein and Jannat-Abad.<sup>19</sup>

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